I. OSE6115 SPRING 2013: HOMEWORK 1, DUE 01/23/2013

(1) Calculate the following convolutions:

1. \( \cos \omega_1 t \otimes \cos \omega_2 t \)

2. \( \cos \omega_1 t \otimes \sin \omega_2 t \)

3. \( \cos \omega_1 t \otimes \text{sinc} \left( \frac{\omega_1}{2\pi t} \right) \)

4. \( \exp \left( -\frac{x^2}{2\sigma^2} \right) \otimes \exp \left( i \frac{x^2}{2\sigma^2} \right) \)

5. \( \exp \left( -\frac{x^2+y^2}{2\sigma^2} \right) \otimes \exp \left( i \frac{x^2+y^2}{2\sigma^2} \right) \)

Plot qualitative figures for the results.

(2) Find the Fourier transform, and plot it qualitatively, for the function

\[
f(t) = \sum_{m=-10}^{10} \text{rect} \left( \frac{t - m T_o}{\tau} \right),
\]

where (a) \( T_o = 4\tau \), (b) \( T_o = 2\tau \), and (c) \( T_o = \tau \).

(3) Find and plot the Fourier transforms of the following functions:

1. \( f(x) = \text{rect} \left( \frac{x-d}{a} \right) + \text{rect} \left( \frac{x+d}{a} \right) \), with (i) \( d = 10a \) and (ii) \( d = 2a \).

2. \( f(x, y) = \left\{ \text{rect} \left( \frac{x-d}{a} \right) + \text{rect} \left( \frac{x+d}{a} \right) \right\} \text{rect} \left( \frac{y}{b} \right) \), with \( d = 10a \), \( b = 20a \).

3. \( f(x, y) = \left\{ \text{rect} \left( \frac{x-d}{a} \right) + \text{rect} \left( \frac{x}{a} \right) + \text{rect} \left( \frac{x+d}{a} \right) \right\} \text{rect} \left( \frac{y}{b} \right) \), with \( d = 10a \), \( b = 20a \).

4. \( f(t) = \sum_{n=0}^{\infty} r^n g(t - nT) \), where \( r \) and \( T \) are constants, \( |r| < 1 \), and the Fourier transform of \( g(t) \) is \( G(\omega) \).

(4) Calculate the convolution of \( f(x) = \text{rect} \left( \frac{x}{a} \right) \) with \( g(x) = \exp \left( i \frac{x^2}{2\sigma^2} \right) \) for \( \left( \frac{a}{\sigma} \right)^2 = 10, 1, 0.5, 0.1 \). You will need to write a computer program (in MATLAB, for example) to do this calculation.