Consider an optical beam that has the following field distribution in some plane:
\[ E(x, y) = A e^{-\frac{x^2 + y^2}{2\sigma^2}} \]

The field is located at the input to the following optical systems. What is the output intensity in x and y?

1. \( d_1 = 0, \ d_2 = f \)
2. \( d_1 = f, \ d_2 = f \)
3. \( d_1 = 2f, \ d_2 = 2f \)
4. \( d_1 = 3f, \ d_2 = \frac{3}{2} f \)
5. \( d_1 = \frac{3}{2} f, \ d_2 = \frac{3}{2} f \)

Assume there is no aperture at the lens.

Consider the system shown below, a single-lens imaging system. Assume the lens has a circular aperture with radius \( a = f/4 \). A point of light is emitted on the optical axis at a distance 2f from the lens.
What is the transverse intensity distribution at:

(a) \( z = 2f \)

(b) \( z = 2f \pm 0.01f \)

What is the intensity distribution along the optical axis, i.e. when \( x = y = 0 \) centered at \( z = 2f \)?

3. (Challenging)

Consider the following system:

The distances \( d_1 \) and \( d_2 \) do not satisfy the imaging equation necessarily. Write the field at the output \( E_{\text{out}}(x, y) \) in terms of:

- the field at the input \( E_{\text{in}}(x) \) in the following form:

\[
E_{\text{out}}(x) = A \int_{-\infty}^{\infty} dx' E_{\text{in}}(x') e^{\frac{k(x^2+x'^2)}{2f_1 \tan \phi}} e^{-i \frac{k x x'}{f_1 \sin \phi}}
\]

Where \( f_1 \) is a constant having units \( d_1 \) length, and \( \phi \) is some fixed angle. Find \( f_1 \) and \( \phi \) in terms of \( f, d_1, d_2, \) and \( f \).

Consider now a cascade of two systems 'a' and 'b', each similar to the above. System 'a' has the parameters \( d_1^a, d_2^a, f^a \); system 'b' has the parameters \( d_1^b, d_2^b, f^b \). Write the transfer between input and output of each system as above, so that now we have.
parameters $f_i$ and $\phi^a$ for system 'a', and $f_i$ and $\phi^b$ for system 'b'.

Define $P^a = \frac{2\phi^a}{\pi}$ and $P^b = \frac{2\phi^b}{\pi}$. Prove that the system resulting from cascading systems 'a' and 'b' back-to-back may also be represented in the same form as above (i.e. the input and output of the full system are related using the above equation), with the new parameter $P_{total} = \frac{2\phi_{total}}{\pi} = P^a + P^b$.

- What is $P^a$ if $d_1 = d_2 = f$? What is $P^a$ if $d_1 = d_2 = 2f$?

- Apply the above results to the special case of the cascade of 2 2-f systems to produce a 4-f system.